Azonosító jel: | | | | | | | | | | | | | | | |

ÉRETTSÉGI VIZSGA • 2023. október 17.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

minden vizsgázó számára

2023. október 17. 8:00

Időtartam: 300 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

OKTATÁSI HIVATAL

Instructions to candidates

1. The time allowed for this examination paper is 300 minutes. When that time is up, you will have to stop working.

2. You may solve the problems in any order.

3. In part II, you are only required to solve four of the five problems. **When you have finished the examination, enter the number of the problem not selected in the square below**. *If it is not clear* for the examiner which problem you do not want to be assessed, the last problem in this examination paper will not be assessed.

4. On solving the problems, you may use a calculator that cannot store and display textual information. You may also use any edition of the four-digit data tables. The use of any other electronic device or printed or written material is forbidden!

5. **Always write down the reasoning used to obtain the answers. A major part of the score will be awarded for this.**

6. **Make sure that calculations of intermediate results are also possible to follow.**

7. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**

8. On solving the problems, theorems studied and given a name in class (e.g. the Pythagorean Theorem or the height theorem) do not need to be stated precisely. It is enough to refer to them by name, but their applicability needs to be briefly explained. Reference to other theorems will be fully accepted only if the theorem and all its conditions are stated correctly (proof is not required) and the applicability of the theorem to the given problem is explained.

9. Always state the final result (the answer to the question of the problem) in words, too!

10. Write in pen. Diagrams may be drawn in pencil. The examiner is instructed not to mark anything written in pencil, other than diagrams. If you cancel any solution or part of a solution by crossing it over, it will not be assessed.

11. Only one solution to each problem will be assessed. In case of more than one attempt to solve a problem, **indicate clearly** which attempt you wish to be marked.

12. Please, **do not write in the grey rectangles**.

I.

1. **a)** Solve the following equation where x and y are **positive integers**.

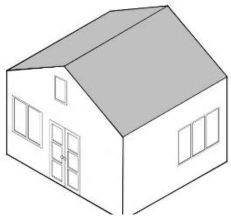
$$\frac{x}{8} = \frac{1.5}{y}$$

b) Solve the following equation over the set of real numbers.

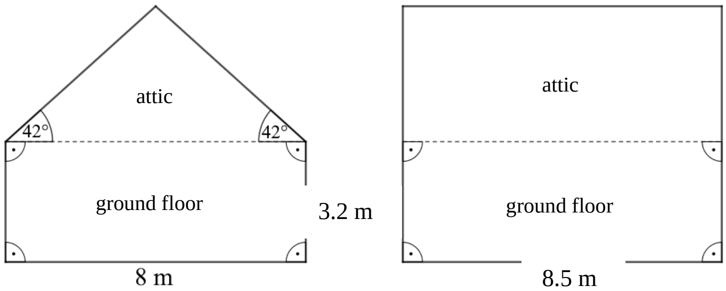
$$3 \cdot 9^{x} - 3^{x+3} = 3^{x} - 9$$

a)	4 points	
b)	7 points	
T.:	11 points	

2. The shape of a family home is that of a cuboid with a matching triangular straight prism for an attic. Some of the measures of this house are shown below in the front and side view diagrams. (Ignore the thickness of the walls.)



illustration



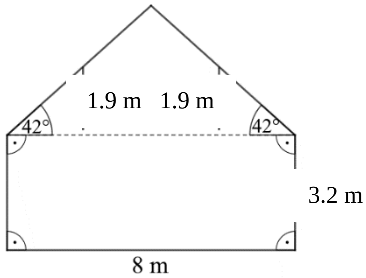
Use the above measures to answer the following questions.

a) The full roof surface is covered with tiles. How much is this surface?
Give your answer in m^2, rounded to the nearest integer.

b) How many cubic metres is the full volume of the house (including both the attic and the ground floor)?

The area of the attic is legally declared as living space only where there is a minimum height of 1.9 metres.

c) How many square metres of living space are there in this house altogether (both in the attic and on the ground floor)?



a)	4 points	
b)	4 points	
c)	4 points	
T.:	12 points	

3. Tomi's soccer trainer evaluates the performance of each of his players after each game on a scale of 1-10 (integers only). Tomi's scores on the first seven games of the season were 6, 8, 6, 2, 8, 8, 6.

a) Calculate the mean and the standard deviation of these seven scores.

After the next three games it turned out that Tomi's mean of the ten scores was 6.3, the range was 8 and the data had a single mode.

b) Determine the score Tomi got on each of these three games.

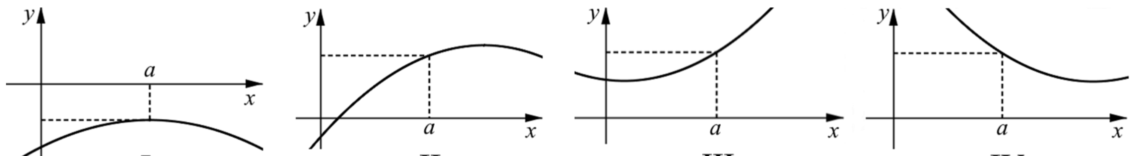
After he got his score on the 11^{th} game, Tomi's average decreased by a tenth of this last score, compared to the 6.3-point average he had after the first 10 games.

c) What score did Tomi get on his 11^{th} game?

a)	3 points	
b)	7 points	
c)	3 points	
T.:	13 points	

4. The quadratic function *f* is defined over the set of real numbers. It is known that, at a particular place $a \in \mathbf{R}$, $f(a) > 0$, $f'(a) > 0$ and $f''(a) > 0$ are all true.

a) The diagrams below show the graphs of four quadratic functions. Use these diagrams to fill in the table with *true* or *false* and determine which of the four graphs represent the function *f*. (You do not have to explain your answers at this point.)



I. II. III. IV.

graph	the function value is positive at *a*	the value of the first derivative is positive at *a*	the value of the second derivative is positive at *a*
I.		*false*	
II.			
III.			
IV.			

The graph of function *f* is … .

b) The value of the quadratic function *g* at $x \in \mathbf{R}$ is determined by the expression $g(x) = px^2 + qx + r$ ($p, q, r \in \mathbf{R}$, $p \neq 0$). Determine the values of *p*, *q* and *r* such that $g(1) = 1$, $g'(1) = 2$ and $g''(1) = 4$.

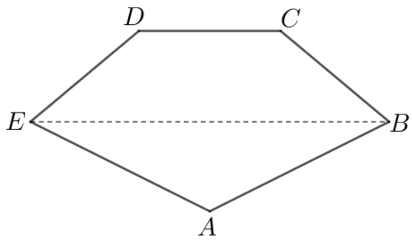
c) Calculate the value of $\int_{-3}^{2} \left(\frac{1}{2}x^2 - 2x + 1 \right) dx$.

a)	6 points	
b)	6 points	
c)	3 points	
T.:	15 points	

II.

You are required to solve any four out of the problems 5 to 9.
Write the number of the problem NOT selected in the blank square on page 2.

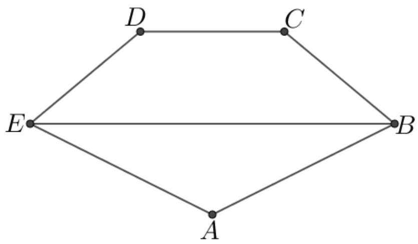
5. In the convex pentagon *ABCDE* $AB = AE = 20$ cm and $BC = CD = DE$. Quadrilateral *BCDE* is a cyclic trapezium that has a 40° interior angle at vertex *B*. The distance between vertex *A* and diagonal *EB* is 10 cm.



a) Give the measure of each (interior) angle of the pentagon.

b) Calculate the area of the pentagon.

c) How many different walks are possible in the 5-point graph *ABCDE* shown in the diagram, given that every edge must be walked exactly once?
(The start of the walk is one of the vertices of the graph. From any one vertex we may only walk along an edge that starts from that vertex.)



a)	4 points	
b)	8 points	
c)	4 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9.
Write the number of the problem NOT selected in the blank square on page 2.

6. A group of six friends, Attila, Boróka, Csaba, Dóra, Emil and Fanni form three teams. Each team consists of two members and each friend is member of a single team only.

a) Prove that there are 15 different possible ways to form the teams.
(Two setups are considered different if there is at least one person who teams up with a different partner in each of them.)

b) Assuming the three teams are formed randomly, calculate the probability that each team will consist of one boy and one girl. (Attila, Csaba and Emil are boys, Boróka, Dóra and Fanni are girls.)

In the end, Attila teamed up with Boróka, Csaba with Dóra and Emil with Fanni. Members of the teams play one-on-one tabletennis games with one another. Both members of each of the three teams play one game against each member of each of the other 2 teams. (Members of the same team do not compete against one another.) Games are played sequentially, one after the other. After a particular game Attila notices that the other five players each have a different number of games played up to that point.

c) How many games has Boróka played so far?

a)	5 points	
b)	4 points	
c)	7 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9.
Write the number of the problem NOT selected in the blank square on page 2.

7. The following formula is used to predict the expected height (in metres) of a certain pine tree:

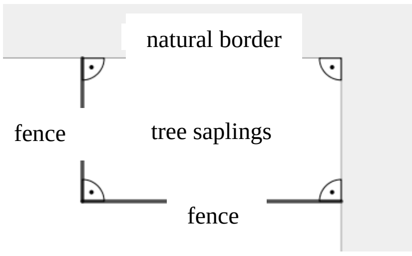
$h(t) = \dfrac{30}{1+59\cdot 0{,}905^{t}}$, where t is the time elapsed since the beginning of the observation, in years.

a) How tall was the tree at the beginning of the observation?

b) How many years after the beginning of the observation will the tree be 10 metres tall?

c) Calculate the limit of the sequence $\{a_n\}$ where $a_n = \dfrac{30}{1+59\cdot 0{,}905^{n}}$.

A rectangular plot of land is separated to grow special tree saplings. There are natural borders along two adjacent sides of the rectangle, so only the other two sides need to be fenced off. Building costs are different on these two sides: on one side it is 5 thousand Ft/m, while on the other side it is 10 thousand Ft/m. The budget for building the fence is 400 thousand Ft.

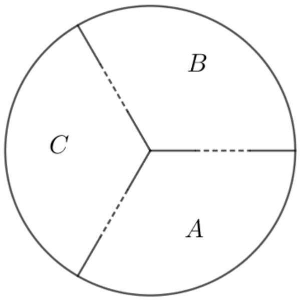


d) How long should each of these fenced sides be to maximise the area of the rectangular plot of land if the budget is not to be exceeded?

a)	2 points	
b)	5 points	
c)	3 points	
d)	6 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9.
Write the number of the problem NOT selected in the blank square on page 2.

8. In a computer game, a circular region is divided into three sections (*A*, *B*, *C*), as seen in the diagram below. There is a passage between any two regions (marked by the dotted lines in the diagram).
Each of these passages are open at a probability of p, independent of one another ($0 < p < 1$). Only an open passage (or passages) grant access to another section.



Let the event E_0 be that it is impossible to move from section A to any other section.

a) Show that the probability of event E_0 is $1 - 2p + p^2$.

b) How much should the value of p be, if the probability of E_0 must be 0.01, at most?

Let the event E_1 be that exactly one other section is accessible from A (not necessarily directly). Let the event E_2 be that both other sections are accessible from A (not necessarily directly).

c) Prove that the probability of event E_1 is $2p - 4p^2 + 2p^3$, while the probability of event E_2 is $3p^2 - 2p^3$.

d) Determine the value of p that maximises the probability of event E_1 and also calculate this maximum probability of event E_1.

a)	3 points	
b)	3 points	
c)	5 points	
d)	5 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9.
Write the number of the problem NOT selected in the blank square on page 2.

9. Use the numbers 2, 4, 6, 8, 10 to create all possible products of two factors, where the first factor is smaller than the second. All such products are then added.

a) Calculate the value of this sum.

Let k be an arbitrary positive integer, greater than 1. Let S_k be the following sum: the numbers 1, 2, 3, … , k (the first k positive integers) are used to create all possible products of two factors, where the first factor is smaller than the second. All such products are then added.

b) Prove that $S_{k+1} = S_k + \frac{k(k+1)^2}{2}$.

c) Prove (by mathematical induction, or otherwise) that for any n integer greater than 1 $S_n = \frac{(n-1)n(n+1)(3n+2)}{24}$.

a)	4 points	
b)	4 points	
c)	8 points	
T.:	16 points	

	Number of problem	score			
		maximum	awarded	maximum	awarded
Part I	1.	11		**51**	
	2.	12			
	3.	13			
	4.	15			
Part II		16		**64**	
		16			
		16			
		16			
		← problem not selected			
Total score on written examination				**115**	

date

examiner

	pontszáma **egész számra** kerekítve	
	elért	programba beírt
I. rész		
II. rész		

dátum

dátum

javító tanár

jegyző